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Lift-Dispersion Control of Spinning Re-entry Vehicles

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A control system is described that will limit cross-range dispersion of spinning re-entry vehicles caused by lift variations. The controls consist of small trim flaps or reaction jets that utilize information derived from strapdown motion sensors to minimize lift nonaveraging. Because epicyclic oscillations in angle of attack do not cause dispersion, only the phasing of complex angle-of-attack motion must be controlled to minimize the net transverse velocity that would otherwise result from lift variations. This requires relatively little control energy compared with a system that would trim the vehicle at zero angle of attack. A simple control law is derived that specifies minimum feedbacks required, and the open- and closed-loop vehicle response to impulse disturbance moments is demonstrated.

Nomenclature

a,b,c,d = feedback gains
a_1,b_1,c_1,d_1 = feedback gains
A_0,A_1,A_2,A_3 = coefficients of characteristic equation
A_N = complex normal acceleration $= A_y + iA_z$
A_y = y component of normal acceleration
A_z = z component of normal acceleration
C_{L_α} = lift force derivative
$C^*_{L_\alpha}$ = damping parameter $= C_{L_\alpha} QS/mu$
$C^*_{m_q}$ = pitch damping parameter $= -C_{m_q} QSd^2/2Iu$
C_{m_α} = pitch moment derivative
d = reference diameter
F = see Eq. (3)
G = see Eq. (4)
$H(t)$ = Heaviside unit step function
i $= \sqrt{-1}$
I = pitch or yaw moment of inertia
I_x = roll moment of inertia
L_α = lift force derivative $= C_{L_\alpha} QS$
m = vehicle mass
m_t = complex trim disturbance moment $= m_{t_y} + im_{t_z}$
m_δ = complex control moment $= m_{\delta_y} + im_{\delta_z}$
m_* = moment impulse
Δm_y = moment step
N_0,N_1,N_2 = coefficients of numerator polynomial
p = roll rate
q = pitch rate
Q = dynamic pressure
r = yaw rate
s = Laplace transform variable
S = aerodynamic reference area (base area)
t = time
u = vehicle velocity
v = component of transverse dispersion velocity
ΔV = complex transverse dispersion velocity $= v + iw$
w = component of transverse dispersion velocity
α = angle of attack
β = angle of sideslip
$\delta(t)$ = unit impulse function
Δ = characteristic, Eq. (14)
η = complex lateral rate $= q + ir$
μ = moment of inertia ratio $= I_x/I$
ν = aerodynamic damping parameter $= C^*_{m_q} + C^*_{N_\alpha}$
ξ = complex angle of attack $= \beta + i\alpha$
ω = undamped natural pitch frequency $= (-C_{m_\alpha} QSd/I)^{1/2}$

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Introduction

SIGNIFICANT dispersion errors can result from lift variations during the re-entry phase of a long-range ballistic re-entry vehicle trajectory. The most notable errors result when the roll rate is driven to zero with the presence of trim asymmetries that cause lift.[1] Such errors can be avoided with roll control by maintenance of a nonzero roll rate during re-entry. Even with a steady, nonzero roll rate, lift variations caused by ablation shape change and asymmetric boundary-layer transition progression can cause moderate re-entry dispersion errors.[2-4] Because the magnitudes of force and moment asymmetries that can cause significant dispersion are relatively small, and because the epicyclic component of angle-of-attack motion does not produce dispersion,[4,5] the motion can be controlled by the use of small pitch and yaw control moments to minimize the net transverse dispersion velocity.

In a previous paper,[6] a method was formulated for control of cross-range dispersion of a spinning missile flying in untrimmed motion. The missile was assumed to be untrimmed as a result of another control loop that modulates the angle of attack for drag control of range error.[7] In the untrimmed condition, the lift vector precesses in space at a characteristic frequency that can be well above the roll rate. The control law for this case differs from that for a vehicle flying in trimmed motion, which is the usual condition of a re-entry vehicle after the initial angle-of-attack error has converged. In trimmed flight, the vehicle is in lunar motion with respect to the mean flight path; i.e., the vehicle executes a spiral motion with the same meridian remaining windward. In this condition, the windward meridian rotation rate is zero and the lift vector precesses in space at a frequency approximately equal to the vehicle roll rate. The control law for such a system is derived in this paper, and the closed-loop response of a re-entry vehicle to simple disturbances is compared with the open loop response. It is shown that control should be possible with information derived from conventional strapdown accelerometers and rate gyros. For the special condition in which the roll rate is maintained at 50% of the undamped natural pitch frequency, control can be achieved with information derived solely from lateral accelerometers. However, this might be difficult to implement because of uncertainty in the aerodynamic pitch frequency and critical roll rate, which could outweigh the advantage of fewer sensors.

Analysis

Control Equations

The equations of motion of a spinning missile in body-fixed coordinates can be written[8]

$$\ddot{\xi}+F\dot{\xi}+G\xi=i(m_t+m_\delta) \tag{1}$$

where

$$\xi=\beta+i\alpha \tag{2}$$

$$F=\nu+ip(2-\mu) \tag{3}$$

$$G=\omega^2-p^2(i-\mu)+ip(\nu-\mu C^*_{L_\alpha}) \tag{4}$$

and m_t and m_δ are defined as the ratio of the complex disturbance and control moments to the pitch moment of inertia, respectively. A complex transverse velocity in the cross plane due to lift can be written[4]

$$\Delta V=v+iw=-\frac{L_\alpha}{m}\int_0^t \xi e^{ip\tau}\mathrm{d}\tau \tag{5}$$

where L_α is the lift force derivative and $p\tau$ is the roll angle, or precession angle of the lift vector for a trimmed vehicle with constant roll rate p. The upper limit of the integral in Eq. (5) is assumed to be sufficiently large to include perturbations in the complex angle of attack that cause dispersion as a result of lift. The transverse velocity is defined in a plane normal to the mean flight path. The rate of trajectory bending is assumed to be small relative to the missile angular rates. The complex control moment m_δ that will minimize the net transverse velocity ΔV caused from some trim disturbance moment m_t is defined as follows. If we resolve Eq. (1) into its real and imaginary components and take the Laplace transform with respect to time, we obtain the coupled linear control equations

$$\begin{bmatrix} s^2+\nu s+\omega^2-p^2(1-\mu) & p(2-\mu)s+p(\nu-\mu C^*_{L_\alpha}) \\ -p(2-\mu)s-p(\nu-\mu C^*_{L_\alpha}) & s^2+\nu s[\omega^2-p^2(1-\mu)] \end{bmatrix}$$

$$\times\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\begin{bmatrix}m_{t_y}\\ -m_{t_z}\end{bmatrix}+\begin{bmatrix}m_{\delta_y}\\ -m_{\delta_z}\end{bmatrix} \tag{6}$$

With the upper limit of the integral in Eq. (5) assumed to be infinitely large, the Laplace transform of the transverse velocity can be written

$$\Delta V(s)=-(L_\alpha/ms)[\beta(s-ip)+i\alpha(s-ip)] \tag{7}$$

where $\alpha(s-ip)$ and $\beta(s-ip)$ are functions of $s-ip$ in which the complex translation results from multiplication by the exponential in Eq. (5). We assume control moments of the form

$$m_{\delta_y}=-a\alpha-b\beta-c\dot{\alpha}-d\dot{\beta} \tag{8}$$

$$m_{\delta_z}=a_1\alpha+b_1\beta+c_1\dot{\alpha}+d_1\dot{\beta} \tag{9}$$

which, when substituted in Eq. (6), give the control equations

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\begin{bmatrix}m_{t_y}\\ -m_{t_z}\end{bmatrix} \tag{10}$$

where

$$A_{11}=s^2+(\nu+c)s+\omega^2-p^2(1-\mu)+a$$

$$A_{12}=[p(2-\mu)+d]s+p(\nu-\mu C^*_{L_\alpha})+b$$

$$A_{21}=[c_1-p(2-\mu)]s-p(\nu-\mu C^*_{L_\alpha})+a_1$$

$$A_{22}=s^2+(\nu+d_1)s+\omega^2-p^2(1-\mu)+b_1$$

Solution for Impulsive Trim

Consider an impulsive trim moment of the form

$$m_{t_y}=m_*\delta(t) \qquad m_{t_z}=0 \tag{11}$$

where $\delta(t)$ is the unit impulse function. The solution to Eq. (10) is then

$$\alpha=(m_*/\Delta)[s^2+(d_1+\nu)s+\omega^2-p^2+b_1] \tag{12}$$

$$\beta=-(m_*/\Delta)[(c_1-2p)s+a_1-\nu p] \tag{13}$$

where

$$\Delta=s^4+A_3s^3+A_2s^2+A_1s+A_0 \tag{14}$$

$$A_3=c+d_1+2\nu \tag{15}$$

$$A_2=2(\omega^2+p^2)+a+b_1+cd_1-c_1d+\nu^2$$
$$+\nu(c+d_1)+2p(d-c_1) \tag{16}$$

$$A_1=2\nu(\omega^2+p^2)+(c+d_1)(\omega^2-p^2)+(a+b_1)\nu+b_1c$$
$$+ad_1-c_1b-\nu p(c_1-d)-2p(a_1-b)-a_1d \tag{17}$$

$$A_0=(\omega^2-p^2)^2+(a+b_1)(\omega^2-p^2)+ab_1$$
$$-a_1b+p(b-a_1)\nu+p^2\nu^2 \tag{18}$$

and the transverse velocity can be written in the form

$$\frac{\Delta V(s)}{-L_\alpha m_*/m}=\frac{N_2s^2+N_1s+N_0}{s(s^4+D_3s^3+D_2s^2+D_1s+D_0)}$$
$$=\frac{N_0/D_0}{s}+\frac{F(s)}{s^4+D_3s^3+D_2s^2+D_1s+D_0} \tag{19}$$

where the inertia ratio μ has been neglected relative to unity for simplicity. It is well known that the epicyclic component of the motion does not cause dispersion.[4,5] Hence, only the residue of the pole at the origin is of interest and the net steady-state transverse velocity increment ΔV_{ss} resulting from the impulse is

$$\frac{\Delta V_{ss}}{-L_\alpha m_*/m}=\frac{N_0}{D_0}=\frac{-a_1+d_1p+2\nu p+i(\omega^2+b_1+c_1p-4p^2)}{A_0+p^4+ip^3A_3-p^2A_2-ipA_1} \tag{20}$$

This velocity will be zero for feedbacks that cause the real and imaginary components of the numerator of Eq. (20) to vanish, which requires that

$$-a_1+d_1p+2\nu p=0 \tag{21}$$

$$\omega^2+b_1+c_1p-4p^2=0 \tag{22}$$

For the assumed feedbacks Eqs. (8) and (9), only feedbacks for m_{δ_z} are required for the disturbance moments assumed in Eq. (11). For system stability, the coefficients of the characteristic equation Eqs. (14) must be positive and

$$A_1(A_3A_2-A_1)-A_3^2A_0>0 \tag{23}$$

It is found from the coefficients, Eqs. (15-18), that these conditions will be satisfied with only the feedback gains a_1 and b_1. Equations (21) and (22) for zero dispersion then require that

$$a_1=2\nu p \tag{24}$$

$$b_1=4p^2-\omega^2 \tag{25}$$

Because the gains defined by Eqs. (24) and (25) require a knowledge of aerodynamic coefficients, it may be desirable to minimize the dependence on aerodynamics by retaining the feedbacks corresponding to the gains c_1 and d_1 in Eqs. (21) and (22). For the special condition when the roll rate is half the pitch frequency ($p=\omega/2$), the gain b_1 is zero, and only a single feedback is required.

Step Trim Change

With a step change in disturbance moment rather than an impulse, i.e.,

$$m_{t_y}=\Delta m_y H(t) \tag{26}$$

where $H(t)$ is the Heaviside unit step function, the solution to Eq. (10) has the form

$$\alpha=(\Delta m_y/s\Delta)\,[s^2+(d_1+\nu)s+\omega^2-p^2+b_1] \tag{27}$$

$$\beta=-(\Delta m_y/s\Delta)\,[(c_1-2p)s+a_1-\nu p] \tag{28}$$

in which the characteristic Δ is unchanged from Eq. (14). A comparison of Eqs. (27) and (28) with Eqs. (12) and (13) indicates the only change in dispersion velocity calculated from Eq. (7) is an additional pole at $s-ip$. This results in a component of transverse velocity proportional to e^{ipt}, which is oscillatory at the roll frequency and produces no net dispersion for steady, nonzero roll rate. The residue of the pole at the origin remains unchanged from Eq. (20), except for a constant of proportionality. Hence, the feedbacks defined by Eqs. (24) and (25) will give zero dispersion.

Open-Loop Response

We can obtain the open-loop response to impulsive and step trim moments by substituting the complex angle-of-attack results Eqs. (12) and (13) or (27) and (28) into Eq. (7) with the feedbacks set equal to zero. For the impulsive trim moment, the net dispersion velocity is

$$\Delta V_{\text{impulse}}=-iL_\alpha m_*/m\omega^2 \tag{29}$$

which is independent of the damping parameter ν. The dispersion occurs in a direction that initially coincides with the negative body z axis when the impulse occurs about the body y axis at time $t=0$. The response to a trim step is

$$\Delta V_{\text{step}}=L_\alpha \Delta m_y/mp\omega^2 \tag{30}$$

which is also independent of the damping parameter. The dispersion for this case occurs in a direction that initially coincides with the positive body y axis about which the trim moment step occurs at time $t=0$. This open-loop result has been derived previously[4] and shows the inverse dependence of dispersion on roll rate, whereas dispersion from an impulsive moment is independent of roll rate. The units in Eqs. (29) and (30) are consistent, because m_* is a moment impulse divided by moment of inertia, which has units of s^{-1}, whereas Δm_y is moment divided by moment of inertia, which has units of s^{-2}.

Optimal Control

A performance index of a system to control ballistic re-entry vehicle dispersion should include, in addition to dispersion, a measure of the control moments required to effect the control. Therefore, a performance index I is defined as

$$I=W_1\Delta V\Delta V^*+W_2\int_0^\infty m_{\delta_y}^2\,dt+W_3\int_0^\infty m_{\delta_z}^2\,dt \tag{31}$$

where ΔV^* is the complex conjugate of the dispersion velocity ΔV, m_{δ_y} and m_{δ_z} are the control moments defined by Eqs. (8) and (9), and W_1, W_2, and W_3 are weighting constants. Because the classical method used to solve for dispersion velocity yields a solution for the transformed values of the control moments in the complex s plane, the integral-square values of these moments in Eq. (31) can be readily obtained by the use of Phillips integrals[9,10] For example, if the control moment $m_{\delta_z}(s)$ obtained from Eq. (9) with the complex angle-of-attack components for an impulsive trim moment, Eqs. (12) and (13), has the form

$$m_{\delta_z}(s)=\frac{B_3s^3+B_2s^2+B_1s+B_0}{A_4s^4+A_3s^3+A_2s^2+A_1s+A_0} \tag{32}$$

then the integral square value of m_{δ_z} in Eq. (31), defined as I_4, has the value

$$I_4=N_4/D_4 \tag{33}$$

where

$$N_4=B_3^2(A_0A_1A_2-A_0^2A_3)+(B_2^2-2B_1B_3)A_0A_1A_4$$
$$+(B_1^2-2B_0B_2)A_0A_3A_4+B_0^2(A_2A_3A_4-A_1A_4^2)$$
$$D_4=2A_0A_4(A_1A_2A_3-A_0A_3^2-A_1^2A_4)$$

The coefficients A_i are defined in terms of the feedback gains in Eqs. (15-18), with $A_4=1$, and $a=b=c=d=0$; the coefficients B_i are defined by

$$B_0=(\omega^2-p^2)^2a_1+\nu pb_1 \tag{34}$$

$$B_1=\nu a_1+2pb_1+(\omega^2-p^2)c_1+\nu pd_1 \tag{35}$$

$$B_2=a_1+\nu c_1+2pd_1 \tag{36}$$

$$B_3=c_1 \tag{37}$$

To achieve optimum control, values of the feedbacks must then be found that minimize the control moment integral I_4 while limiting the dispersion velocity ΔV to an allowable level.

Numerical Examples

Open- and closed-loop responses to three different forms of disturbance moments were obtained from numerical integration of the equations of motion. The three moment

Table 1 Vehicle dynamics characteristics

$m = 2.57$ slugs	$C_{L_\alpha}^{*} = 2.66\ \mathrm{s}^{-1}$
$s = 0.545\ \mathrm{ft}^2$	$p = 9.95$ rad/s
$\mu = 0.0448$	$u = 16.18$ kft/s
$\omega = 196$ rad/s	$a,b,c,d,c_I,d_I = 0$
$L_\alpha = 1.10 \times 10^5$ lb/rad	$a_I = 2\nu p$
$\nu = 5.19\ \mathrm{s}^{-1}$	$b_I = 4p^2 - \omega^2$

forms are as follows: 1) a moment impulse, 2) a moment step, and 3) a finite duration pulse that consists of stepping the moment of form 2 in the opposite direction after a finite time delay. The missile dynamics characteristics used in the simulations and shown in Table 1 are representative of a long-range ballistic re-entry vehicle in the region of peak re-entry dynamic pressure. Identical control moments that consist of the attitude and rate feedbacks defined in Eqs. (9, 24, and 25) were used in all three cases.

The open- and closed-loop responses to the moment impulse are shown in Figs. 1 and 2, respectively. The impulse was approximated by a rectangular pulse 0.002 s in duration (which is short compared with the period of the highest vehicle natural frequency) and of sufficient magnitude to give a peak open-loop angle-of-attack oscillation amplitude of approximately 1 deg, as shown in Fig. 1a. The open-loop transverse velocity components resulting from the impulse are shown in Figs. 1b and 1c, and the closed-loop vehicle response, for comparison, is shown in Fig. 2. The closed-loop angle-of-attack response is not significantly changed, but the mean value of the transverse velocity w is reduced from approximately 3.5 ft/s to effectively zero, as the theory predicts.

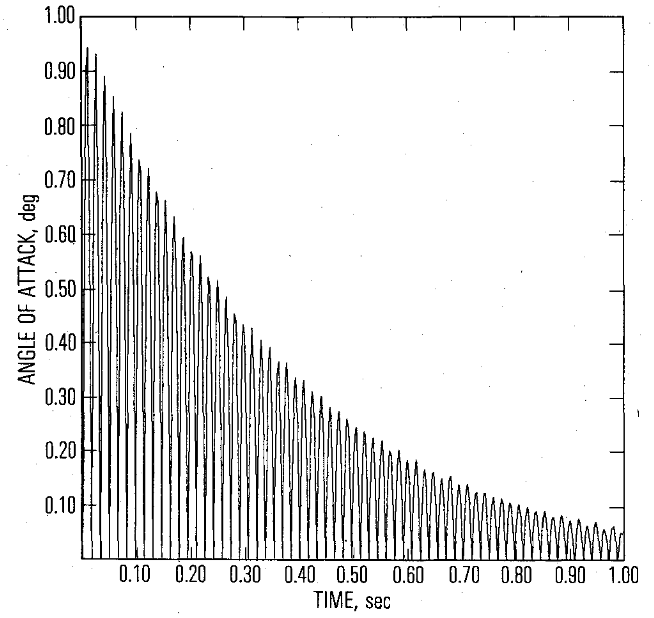
The response to a moment step is shown in Figs. 3 and 4. The missile is initially trimmed at near zero angle of attack, and a trim moment equivalent to 0.5 deg trim angle of attack is applied suddenly at time $t = 0$ and sustained. The open- and closed-loop angle-of-attack responses are shown in Figs. 3a and 4a, and corresponding transverse velocity cross plots are shown in Figs. 3b and 4b. The open-loop dispersion velocity is approximately 38 ft/s whereas the closed-loop velocity has a near-zero mean.

The response to a moment pulse in which the moment in the example of Figs. 3 and 4 is removed at 0.5 s is shown in Figs. 5 and 6. The net open-loop transverse velocity, obtained from the resultant of the velocity components of Fig. 5b, is approximately 46 ft/s, while the closed-loop value from Fig. 6b is approximately 4 ft/s.

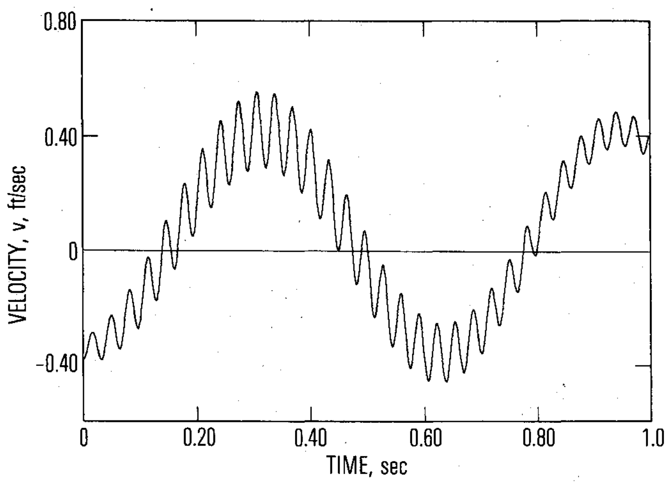
The foregoing results were obtained with the feedback gains a_I and b_I, defined in Eqs. (24) and (25), that, in theory, cause zero dispersion. No consideration was given to optimization of the control moments. If the rate feedback gains c_I and d_I are included in addition to a_I and b_I, we can examine the influence of c_I and d_I on the integral square values of the control moments. The performance index I defined in Eq. (31) was evaluated numerically with different weighting constants to obtain feedback gains that produce minima in the integral square control moments. The results are given in Table 2 where the dispersion index $\Delta V \Delta V^*$ is shown relative to the open-loop value $(\Delta V \Delta V^*)_{\mathrm{ol}}$, and the integral square control moment is shown relative to its value with c_I and $d_I = 0\,(I_{4_0})$.

System Implementation

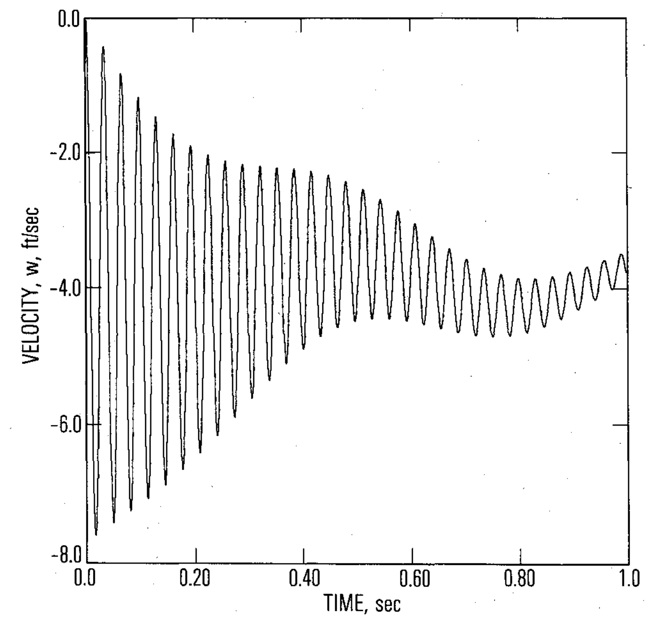
The dispersion velocity to be controlled, as defined in Eq. (5), is an integral of the lateral acceleration in the two orthogonal cross-range inertial directions. This velocity was derived from the lift force expressed as a product of the lift



a) Angle of attack

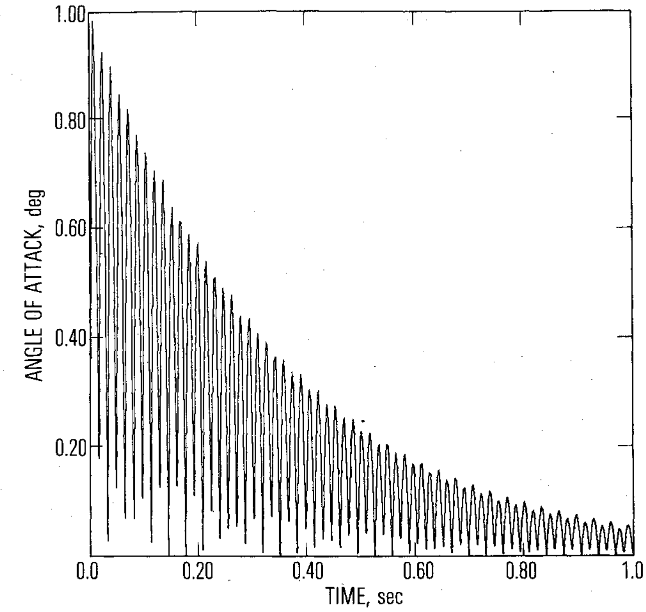


b) ν-component of dispersion velocity

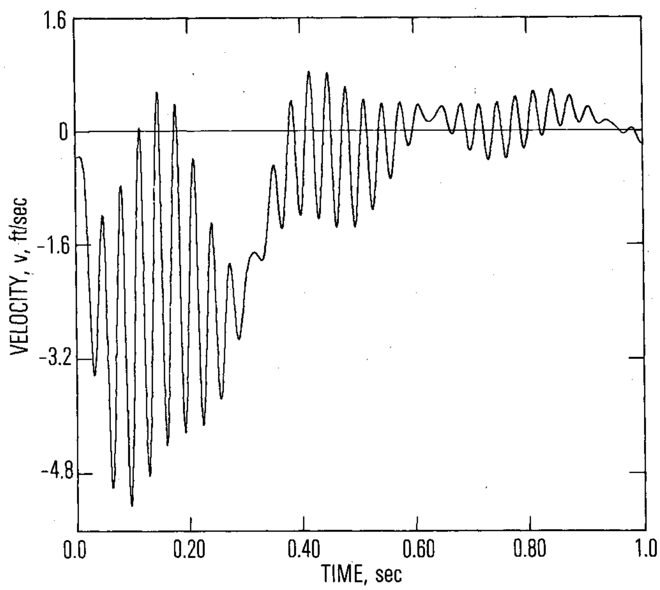


c) ω-component of dispersion velocity.

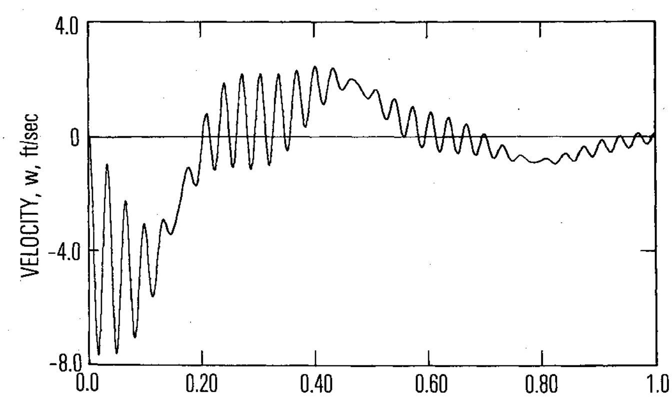
Fig. 1 Open-loop response to moment impulse.



a) Angle of attack

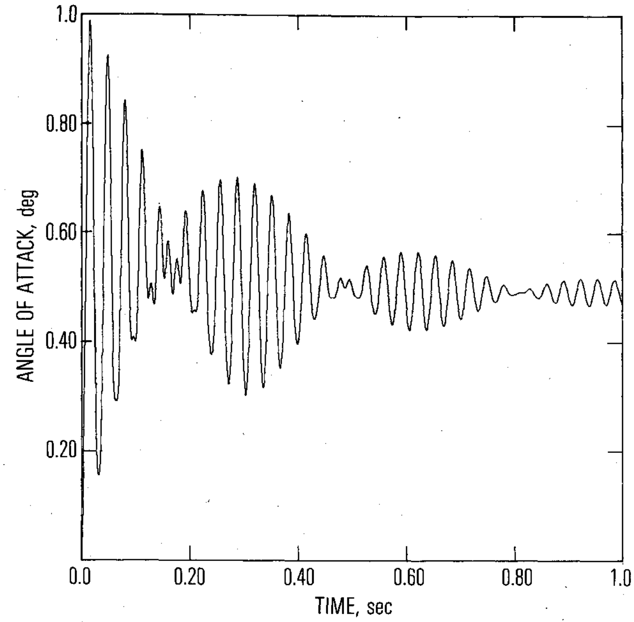


b) v-component of dispersion velocity

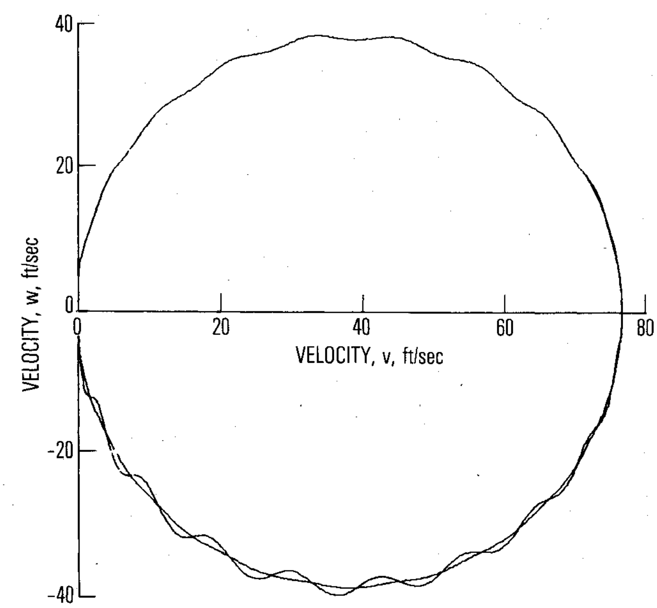


c) ω-component of dispersion velocity

Fig. 2 Closed-loop response to moment impulse.



a) Angle of attack



b) Dispersion velocity crossplot.

Fig. 3 Open-loop response to moment step.

force derivative L_α (assumed to be constant) and the complex angle of attack ξ. Without loss of generality, the quantity $L_\alpha \xi / m$, which is the complex lateral acceleration, can be treated as the control parameter in place of angle of attack, because it can be measured directly with orthogonal lateral accelerometers. The control moments are defined in terms of feedback gains in lateral acceleration and acceleration rate, instead of angle of attack and angle-of-attack rate as defined in Eqs. (8) and (9). The acceleration rates can be obtained either by differentiation of the accelerometer outputs or from rate gyro measurements by use of the relation

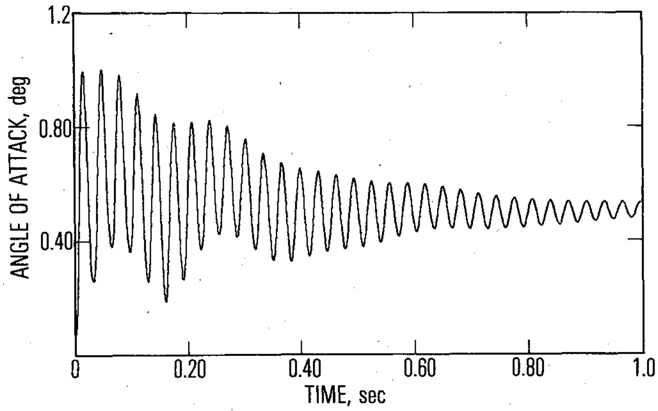
$$\eta = q + ir = -i[\dot{\xi} + (C^*_{L_\alpha} + ip)\xi] \tag{38}$$

between the complex lateral rate η and complex angle of attack ξ. If we define a complex normal acceleration

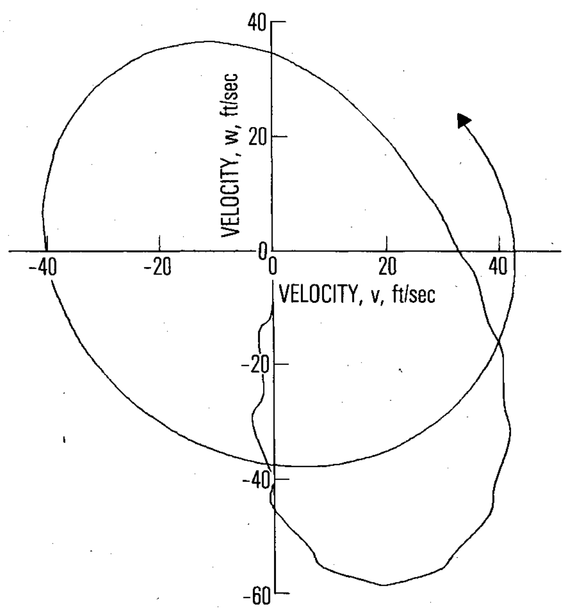
$$A_N = A_y + iA_z \tag{39}$$

Table 2 Influence of feedback gains on performance

Feedback gains				Dispersion index	Control moment index
a_l	b_l	c_l	d_l	$(\Delta V \Delta V^*)/(\Delta V \Delta V^*)_{ol}$	I_4/I_{4_0}
0	0	0	0	1.0	–
1689	−36,570	−44.3	265	0.217	0.366
1946	−37,230	−45.3	216	0.0365	0.435
103	−38,380	0	0	~0	1.0



a) Angle of attack



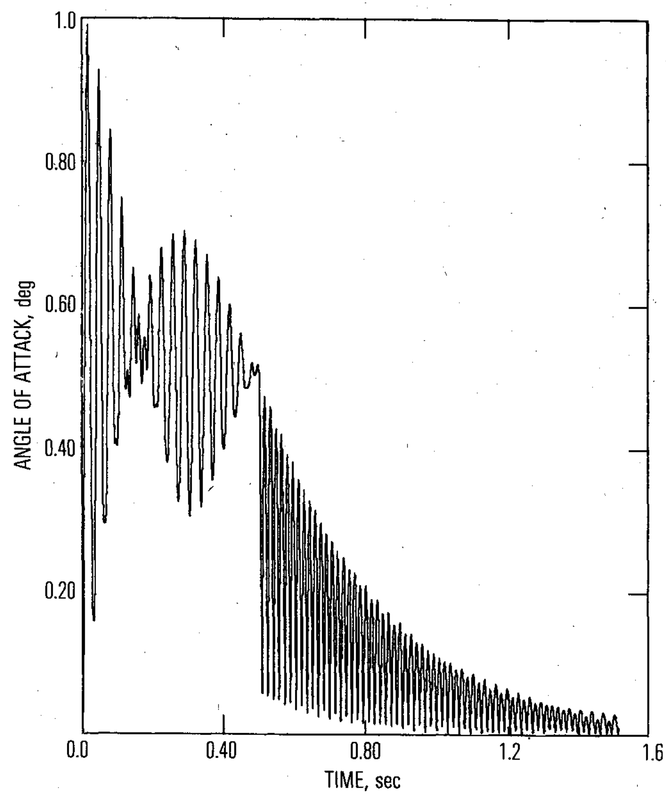
b) Dispersion velocity crossplot.

Fig. 4 Closed-loop response to moment step.

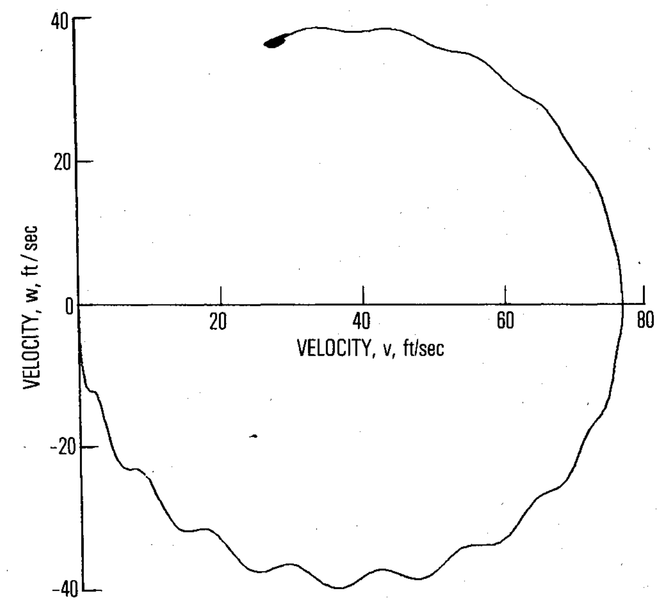
then the acceleration rate $\dot{A}_N$ from Eq. (38) is

$$\dot{A}_N = (iL_\alpha/m)(q+ir) - (C^*_{L_\alpha} + ip)A_N \tag{40}$$

The control moments are generated from small pitch and yaw trim flaps or reaction jets to produce a net control moment in a plane determined from a resolution of the lateral accelerometer measurements. The control system acts as a regulator to maintain a zero value for the net transverse dispersion velocity.

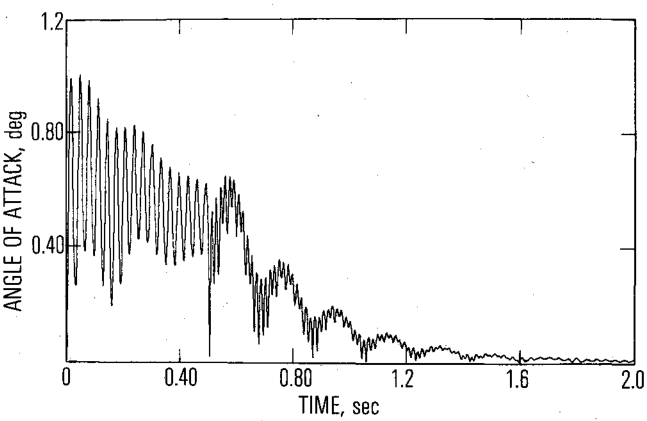


a) Angle of attack

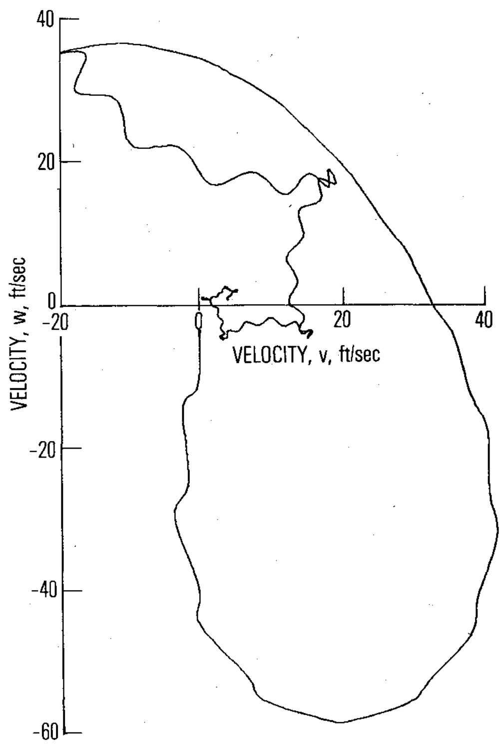


b) Dispersion velocity crossplot

Fig. 5 Open-loop response to moment pulse.



a) Angle of attack



b) Dispersion velocity crossplot.

Fig. 6 Closed-loop response to moment pulse.

Conclusions

A control system has been formulated for limiting cross-range dispersion of a spinning re-entry vehicle caused by lift nonaveraging. The system would use small trim flaps or reaction jets to minimize transverse dispersion velocity based on information derived from body-fixed lateral accelerometers and rate gyros. A simple control law derived from impulsive-type disturbance moments indicates that a fixed set of feedbacks will effectively limit dispersion for different generic moment forms that can otherwise produce large dispersion. For a special condition in which the roll rate is maintained at 50% of the vehicle natural pitch frequency, control can be achieved with information derived entirely from lateral accelerometers. Open- and closed-loop vehicle responses have been demonstrated with digital computer simulations of the equations of motion.

The results were obtained for feedback gains that minimize dispersion without regard for minimization of the control moments required. When the integral square value of the control moments is included in the performance index, with suitable weighting functions on the dispersion and control moments, both the dispersion and control moments can be reduced. The dispersion performance index, defined as the square of the absolute magnitude of the transverse velocity, is reduced to 3.65% of its open loop value when the integral square value of the control moment is reduced to 43.5% of the value required to cause zero dispersion. Similarly, the dispersion index is reduced to 21.7% of its open-loop value when the integral square control moment is reduced to 36.6% of the value required for zero dispersion.

Although the results are derived for the aerodynamic characteristics of a spinning, nominally axisymmetric re-entry vehicle, the control concept should apply to any nominally axisymmetric spinning missile when it is desired to minimize cross-range dispersion caused by lift variations. Such applications might include control of dispersion resulting from in-flight variations in mass asymmetries such as principal axis misalignment and center-of-gravity offset or from variations in trim angle-of-attack asymmetries caused by configurational changes such as ablation asymmetries. Because large control moments are required for relatively small variations in trim angle of attack for re-entry applications, the control moments may have to be generated aerodynamically, e.g., with small trim flaps. For applications that require smaller absolute values of the control moments, control may be achieved with reaction jets. In either case, the control requirements based on the concept of minimizing dispersion velocity should be appreciably less than those for a system that would control cross-range error by maintaining a nominally zero trim angle of attack.

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